LITE : Efficiently Estimating Gaussian Probability of Maximality

Nicolas Menet[1], Jonas Hübotter[1], Parnian Kassraie[1], Andreas Krause[1]

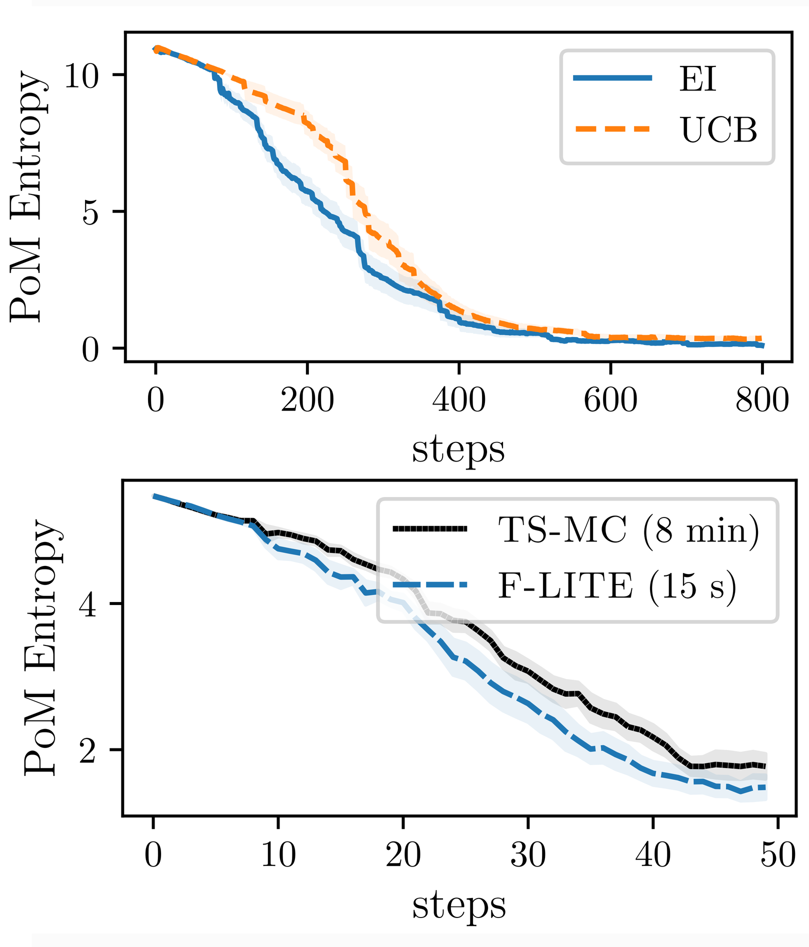
ETH zürich

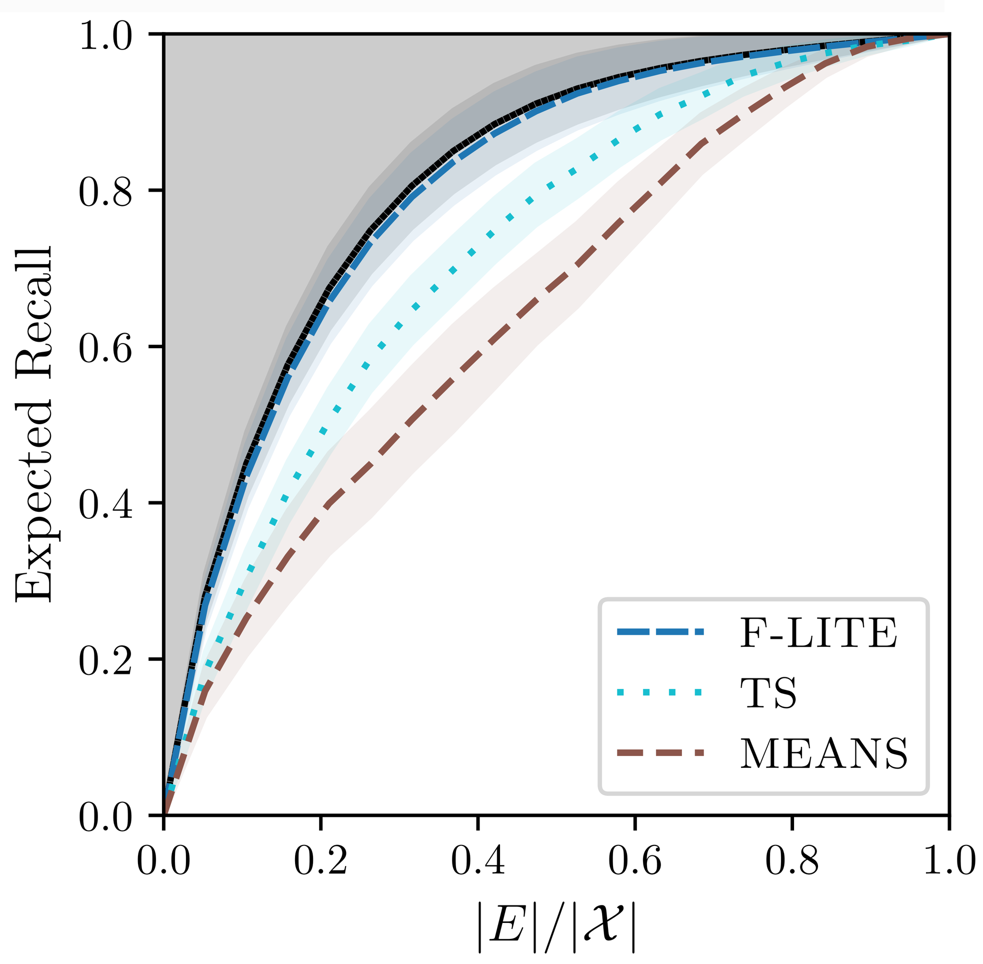
[1] ETH Zürich

Learning & Adaptive Systems

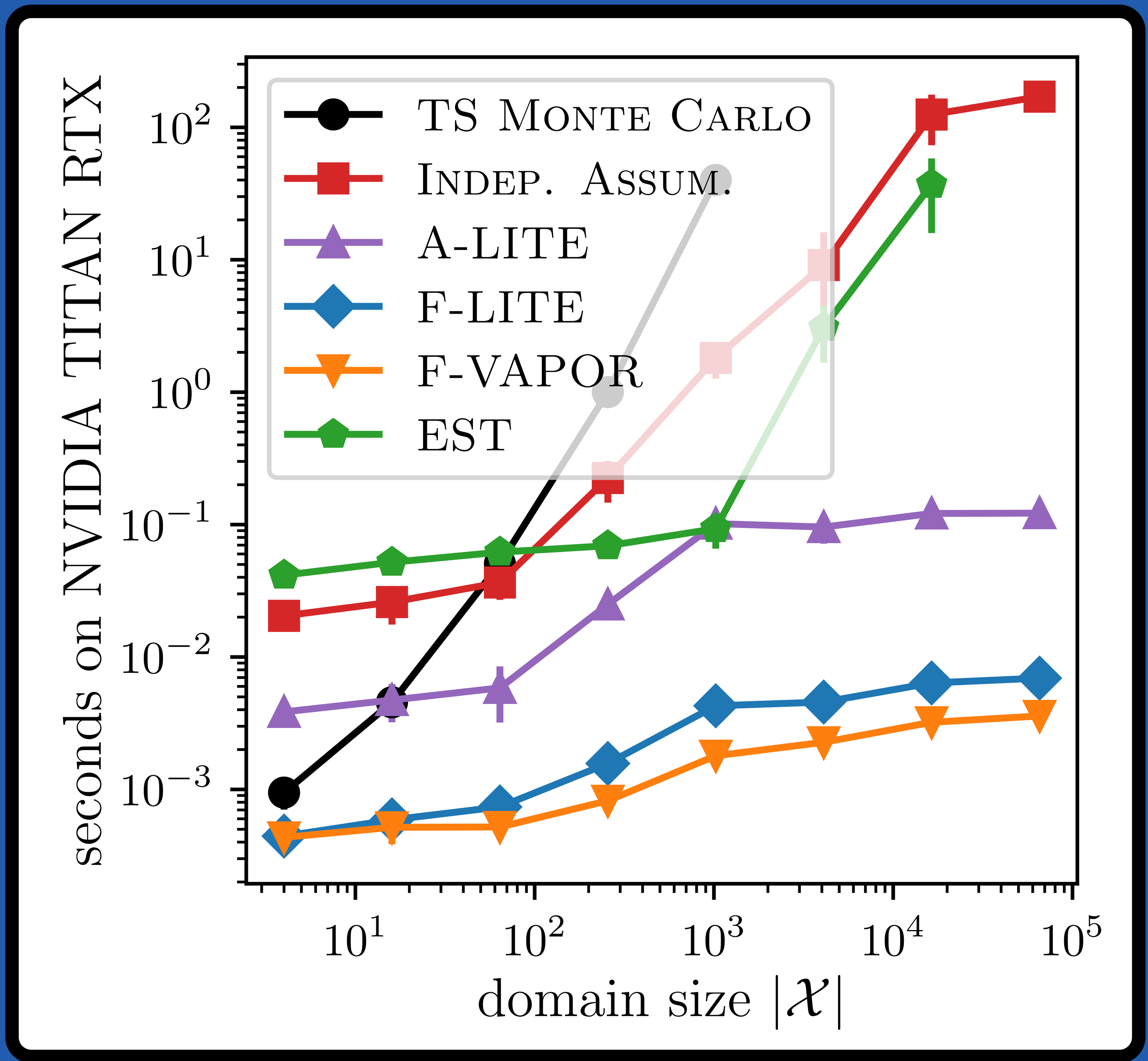
Motivation

The probability of maximality of Gaussian vectors occurs in Thompson sampling, entropy search, entropy estimation, and inverse reinforcement learning, but is poorly understood and very expensive to compute, scaling in $\theta(|\mathcal{X}|^4)$.





An almost-linear time estimator of Gaussian probability of maximality that outperforms prior work in accuracy and runtime.



Contributions

1. By adopting an *independence assumption* on the Gaussian entries, we simplify to a one-dimensional integral.

2. To avoid costly numerical integration, we approximate the integrand, which is a CDF, with the CDF of a standard normal and fit m_x and s_x.

$$\tilde{p}_x = \mathbb{P}[\tilde{F}_z \leq \tilde{F}_x \ \forall z \neq x] = \mathbb{E} \prod_{z \neq x} \mathbb{P}[\tilde{F}_z \leq \tilde{F}_x \mid \tilde{F}_x]. \quad (1)$$

$$\approx \mathbb{E}\Phi\left(\frac{\tilde{F}_x - m_x}{s_x}\right) = \Phi\left(\frac{\mu_{F_x} - m_x}{\sqrt{\sigma_{F_x}^2 + s_x^2}}\right) \quad (2)$$

- A-LITE uses quartile matching to fit the free parameters m_x and s_x.
- F-LITE sets $s_x = 0$ (extreme-value theorem) and uses the normalization condition to find $m_x = \kappa^*$.

Theoretical Insights

Proposition 4. *Define the variational objective*

$$\mathcal{W}(p) := \sum_{x \in \mathcal{X}} p_x \cdot \left(\mu_{F_x} + \underbrace{\sqrt{2\tilde{I}(p_x)} \cdot \sigma_{F_x}}_{\text{exploration bonus}}\right), \quad (5)$$

with the quasi-surprisal $\tilde{I}(u) := (\phi(\Phi^{-1}(u))/u)^2/2$. *Then the maximizer of* $\mathcal{W}$ *among elements of the probability simplex is given by* F-LITE, *i.e., by* q *with*

$$q_x := \Phi\left(\frac{\mu_{F_x} - \kappa^*}{\sigma_{F_x}}\right) \quad \text{with } \kappa^* \text{ s.t. } \sum_x q_x = 1.$$

	Synthetic Distributions	1-dim GP	2-dim GP (E.2)	DropWave (E.3)	Quadcopter
EST	11.54 ± 0.20	45.6 ± 2.7	15.1 ± 1.2	5.17 ± 0.64	14.3 ± 2.0
VAPOR	9.89 ± 0.11	37.0 ± 2.0	15.7 ± 1.0	5.70 ± 0.72	17.2 ± 2.5
F-LITE (ours)	4.65 ± 0.08	$\mathbf{13.7 \pm 1.0}$	10.9 ± 0.7	$\mathbf{4.87 \pm 0.60}$	11.1 ± 1.4
A-LITE (ours)	$\mathbf{3.76 \pm 0.06}$	$\mathbf{14.1 \pm 1.0}$	$\mathbf{7.5 \pm 0.5}$	$\mathbf{4.32 \pm 0.53}$	$\mathbf{8.7 \pm 0.9}$
Indep. Assum.	0.00 ± 0.00	6.7 ± 0.4	6.6 ± 0.2	3.85 ± 0.54	9.0 ± 1.0