Theoretical Computer Science
List of Problems

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Outline

Undecidable problems

NP-complete problems

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Special Halting Problem

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Post's Correspondence Problem (PCP)

given: finite sequence $(x_1, y_1), \ldots, (x_k, y_k)$ where $x_i, y_i \in \Sigma^+$
problem: is there a sequence of indices $i_1, \ldots, i_n \in [k], n > 0$ such that $x_{i_1} \ldots x_{i_n} = y_{i_1} \ldots y_{i_n}$?

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- G ambiguous?
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- for some regular expression α, $L(G) = L(\alpha)$?

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NONEQUIVALENCE

given: two propositional formulas F_1, F_2
problem: is there an assignment $\mathcal{A}$ such that $\mathcal{A}(F_1) \neq \mathcal{A}(F_2)$?

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given: undirected graph G
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TRAVELLING SALESMAN (TSP)

given: matrix $(M_{ij})_{1 \leq i,j \leq n}$ of *distances*, $k \in \mathbb{N}$
problem: is there a roundtrip (Hamiltonian circuit) of length $\leq k$?

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given: $T_1, \ldots, T_n \subseteq M$ with M finite, $k \in \mathbb{N}$
problem: is there $i_1, \ldots, i_n \in [k]$ with $M = T_{i_1} \cup \cdots \cup T_{i_n}$?

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CLIQUE

given: undirected graph G, $k \in \mathbb{N}$
problem: does G have a clique of at least size k?

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KNAPSACK

given: $a_1, \ldots, a_n, b \in \mathbb{N}$
problem: is there $R \subseteq [n]$ with $\sum_{i \in R} a_i = b$?

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BINPACKING

given: *can size* $b \in \mathbb{N}$, *# of cans* $k \in \mathbb{N}$, *objects* $a_1, \dots, a_n \in \mathbb{N}$
problem: can each object be assigned to a can without any can overflowing?