Theoretical Computer Science
List of Problems

Jonas Hübotter

Outline

Undecidable problems

NP-complete problems

Undecidable problems

Special Halting Problem

$K = \{w \in \{0,1\}^* \mid M_w[w] \downarrow\}$

General Halting Problem

$H = \{w\#x \mid w, x \in \{0,1\}^* \wedge M_w[x] \downarrow\}$

Halting Problem on an empty tape

$H_0 = \{w \in \{0,1\}^* \mid M_w[\epsilon] \downarrow\}$

Post's Correspondence Problem (PCP)

given: finite sequence $(x_1, y_1), \ldots, (x_k, y_k)$ where $x_i, y_i \in \Sigma^+$
problem: is there a sequence of indices $i_1, \ldots, i_n \in [k], n > 0$ such that $x_{i_1} \ldots x_{i_n} = y_{i_1} \ldots y_{i_n}$?

Undecidable problems

CFG problems

Let G, G_1, G_2 be CFGs.

- $L(G_1) \cap L(G_2) = \emptyset$?
- $|L(G_1) \cap L(G_2)| = \infty$?
- $L(G_1) \cap L(G_2)$ context-free?
- $L(G_1) \subseteq L(G_2)$?
- $L(G_1) = L(G_2)$?
- G ambiguous?
- G regular?
- G deterministic?
- for some regular expression α, $L(G) = L(\alpha)$?

NP-complete problems

SAT

given: propositional formula F
problem: is F satisfiable?+-

CNF-SAT

given: propositional formula F in kCNF for $k \geq 3$
problem: is F satisfiable?

NONEQUIVALENCE

given: two propositional formulas F_1, F_2
problem: is there an assignment $\mathcal{A}$ such that $\mathcal{A}(F_1) \neq \mathcal{A}(F_2)$?

NP-complete problems

HAMILTON

given: undirected graph G
problem: does G have a Hamiltonian circuit (i.e. a circuit visiting every vertex exactly once)?

TRAVELLING SALESMAN (TSP)

given: matrix $(M_{ij})_{1\leq i,j\leq n}$ of *distances*, $k \in \mathbb{N}$
problem: is there a roundtrip (Hamiltonian circuit) of length $\leq k$?

NP-complete problems

COL

given: undirected graph G, $k \geq 3$
problem: is there a vertex coloring with k colors such that no two adjacent vertices are assigned the same color?

SETCOVER

given: $T_1, \ldots, T_n \subseteq M$ with M finite, $k \in \mathbb{N}$
problem: is there $i_1, \ldots, i_n \in [k]$ with $M = T_{i_1} \cup \cdots \cup T_{i_n}$?

CLIQUE

given: undirected graph G, $k \in \mathbb{N}$
problem: does G have a clique of at least size k?

NP-complete problems

KNAPSACK

given: $a_1, \ldots, a_n, b \in \mathbb{N}$
problem: is there $R \subseteq [n]$ with $\sum_{i \in R} a_i = b$?

PARTITION

given: $a_1, \ldots, a_n \in \mathbb{N}$
problem: is there $I \subseteq [n]$ with $\sum_{i \in I} a_i = \sum_{i \notin I} a_i$?

BINPACKING

given: *can size* $b \in \mathbb{N}$, *# of cans* $k \in \mathbb{N}$, *objects* $a_1, \ldots, a_n \in \mathbb{N}$
problem: can each object be assigned to a can without any can overflowing?